Correction method of image distortion of fisheye lens

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Abstract: A new method to correct the image distortion of a fisheye lens was presented. The relationship of radial position of the chief ray at a projection plane and image plane was obtained by tracing the chief ray through the optical system, then a Fourier series was applied to fit the relation curve. Finding the inverse function of the expression of the series, the image without distortion can be obtained in terms of the resultant distorted image. Correction of image distortion of two fisheye lenses was simulated numerically with the method, experiments of distortion correction of two pictures taken with one of fisheye lenses, Nikon 16 mm/*F*2.8 lenses, were performed. The results show that residual radial height error of the restored image to the object is quite small (less than 0.25%) when the projection plane coincides with the object plane. The experiments also demonstrate that the method is feasible.

Key words: fisheye lens; distortion correction; ray tracing; pupil spherical aberration

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鱼眼镜头图像畸变的校正方法

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摘 要: 提出了一种校正鱼眼镜头图像畸变的新方法。通过追迹光学系统的主光线,获得它在投影平面上的径向位置与它在像平面上位置之间的关系曲线;然后用傅立叶级数拟合该关系曲线;并求出该级数的逆函数,结果就可以根据畸变图像复原没有畸变的图像。用该方法数值模拟两个鱼眼镜头系统的图像畸变校正;并用其中一个鱼眼镜头(尼康 16 mm/F2.8)拍摄的两张图像进行了畸变校正实验。结果表明,当设置投影平面和物平面重合时,恢复的图像相对于物体的残余径向高度误差非常小(小于 0.25%)。实验证明了该方法是可行的。

关键词: 鱼眼镜头; 畸变矫正; 光线追迹; 光瞳球差

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0 Introduction

In many applications of photography, such as machine vision, security monitoring, medical diagnostics, and the projection reality and so on, an imaging lens with very large field angle is often required. A fisheye lens usually satisfies the demands, since it has the field angle of 180° or even larger, and does not need stitching of several pictures without the blind area. However, very serious image distortion exists. In many applications, it is necessary to correct the image distortion of fisheye lens.

Many authors have studied the method of correcting image distortion of fisheye lens as well as image restoration algorithms[1-5]. For example, distortion correction of image of fisheye lens based on calibration is often applied[4]. The calibration process needs to take multiple images of the accurate calibration plate from different angles, in order to determine the coordinates of corner points. The calibration process is relatively cumbersome.

In this paper, we develop a method of distortion correction of image of fisheye lens based on tracing of the chief ray without the cumbersome process of calibration. It only needs the parameters of lens. In the next section, we first introduce the projection model of image of a fisheye lens, then derive the relationship of pixel position between the projection plane and the image plane, and find the image distribution on the projection plane. In section two, we make numerical simulations of distortion correction of two fisheye lenses and process two pictures taken with a Nikon 16 mm/F2.8 fisheye lens to validate our method.

1 Principle

1.1 Projection mode of image

Figure 1 shows the projection model of image of a fisheye lens. The chief ray from an object point $P(x_0, y_0)$ intersects a projection plane $x''o''y''$ at $P''(x'',y'')$. Passing through the fisheye lens, it intersects the image plane $x'o'y'$ at $P'(x',y')$. Both the projection plane and the image plane are perpendicular to optical axis. θ_1, θ_2 are the orientation angle of the chief ray, and ω_0, ω_1 the field angle at the object and image space, correspondingly.

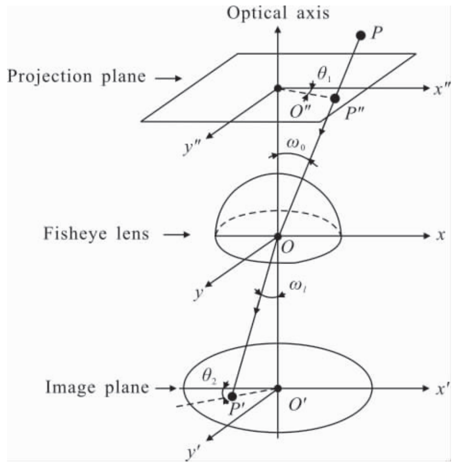


Fig.1 Imaging projection model of fisheye lens

Generally, tangential distortion is much smaller than the radial distortion and can be ignored[6]. Thus we have $\theta_1=\theta_2$. Tracing the chief ray through the optical system of fisheye lens will yield the relation $\omega_i=f(\omega_0)$. According to the position of the projection plane and image plane, then we can get $h_i=f(h_0)$, where $h_i=\overline{O'P'}$, $h_0=\overline{O''P''}$. Finding its reverse function, we can obtain a perspective projection of object on the projection plane. Since the projection screen is a plane, the field angle ω_0 should be limited to be less than 90°. However, if the range of field angle $2\omega_0 \geqslant 180°$, a projection surface of sphere or some curved shape should be applied.

To obtain a pure perspective image, imaging needs single-viewpoint projection, which requires that the chief ray of different field angles passes through the entrance pupil of fisheye lens. However, the pupil spherical aberration $L(\omega_0)$ increases as the increasing field angle, leading to the chief ray to deviate from the ideal direction which satisfies the single-viewpoint projection, as shown the dashed line in Fig.2; their corresponding projection position on the projection plane, P'' and $\overline{P''}$, does not coincide, the displacement is usually very small as $L(\omega_0)$ is much smaller than the object distance. The smaller the distance of the projection plane from P, the smaller the deviation amount of the chief ray. Therefore, it is desirable to set the projection plane close to the region of scene with larger field angle or to the interested region.

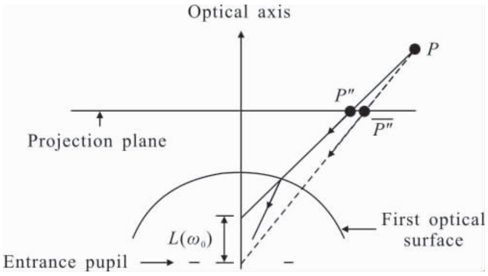


Fig.2 Optical scheme of the pupil spherical aberration

1.2 Relationship between projection and image

In a fisheye lens, the arrangement of lens is rotationally symmetric; moreover, the distortion of image is field-dependent aberration, so that it can be determined by tracing the chief ray through the optical system on the meridional plane[7-8].

Figure 3 shows the optical scheme of a chief ray passing through optical surfaces i and $i+1$, which intersect the optical axis at D_i and D_{i+1}. The chief ray $M_{i-1}O_iO_{i+1}$ impinges the two surfaces at O_i and O_{i+1}; the dot-dashed lines O_iC_i, $O_{i+1}C_{i+1}$ are the normal of them at O_i and O_{i+1}, respectively. ω_{i-1}, ω_i mean the field angle of surface i in object and image space. The angles of incidence and refraction at surface i are denoted with α_i and β_i. r_i, in the following discussions, d_i mean the curvature radius of surface i and the spacing between optical surface i and $i+1$. The refractive index in the object and image space of optical surface i are denoted with n_{i-1} and n_i, respectively.

The sign of the above angles is assumed to be identical to that of the slope of the chief ray with respect to the optical axis or the normal to optical surface correspondingly. The sign of axial distance parameters, including the curvature radius of optical surface, is assumed to be positive if they lie on the right side of the optical surface.

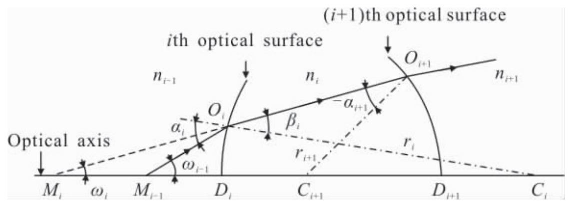


Fig.3 Optical scheme of a chief ray passing through optical surfaces

To the triangle $\Delta M_iO_iC_i$ in Fig.3,

$$\overline{M_iC_{i+1}}=r_{i+1}+d_i-r_i+\frac{\sin\beta_i}{\sin\omega_i}r_i \tag{1}$$

Similarly, to the triangle $\Delta M_iO_{i+1}C_{i+1}$,

$$\sin\alpha_{i+1}=\frac{\overline{M_iC_{i+1}}}{r_{i+1}}\sin\omega_i \tag{2}$$

Substituting Eq.(1) for Eq.(2),

$$\sin\alpha_{i+1}=\frac{r_{i+1}+d_i-r_i}{r_{i+1}}\sin\omega_i+\frac{r_i}{r_{i+1}}\sin\beta_i \tag{3}$$

with

$$\omega_i=\omega_{i-1}+\beta_i-\alpha_i \tag{4}$$

where $\beta_i=\arcsin\left(\frac{n_{i-1}\sin\alpha_i}{n_i}\right)$.

To the first optical surface, setting $i=0$, we have

$$\sin\alpha_1=\frac{r_1+d_0}{r_1}\sin\omega_0 \tag{5}$$

where $d_0=\overline{M_{-1}D_1}$, as shown in Fig.4.

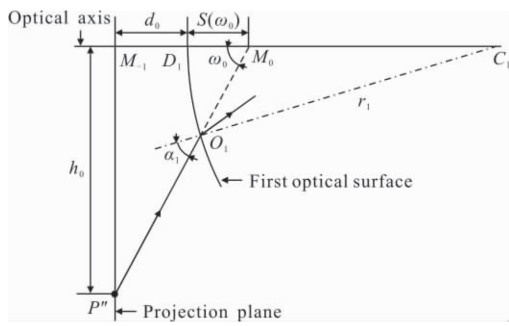


Fig.4 Optical scheme of the chief ray though the projection plane and passing through the first optical surface

Equations.(3) to (5) are the transfer equations of the chief ray. If the optical parameters of a fisheye lens and the field angle in object space ω_0 are given, the above equations can then be used to calculate the field angle in the image space of the last optical surface ω_l. Tracing many field angles, then the relation curve between ω_l and ω_0 can be obtained.

If a projection plane is positioned at a finite distance of d_0 from the first optical surface, the chief ray will intersect the projection plane at a distance of h_0 from the optical axis, as shown in Fig.4. The dot-dashed line O_1C_1 is the normal of the surface at O_1, then h_0 is determined:

$$h_0=(S(\omega_0)+d_0)\tan\omega_0 \tag{6}$$

where $S(\omega_0)$ means the distance of the intersection point of the chief ray to the optical axis from the first optical surface, which varies with the field angle. For a fisheye lens it can be determined by the reverse tracing of the chief ray from the aperture stop with Eqs.(3) to (5)[6].

Figure 5 shows the chief ray passing through the last optical surface and intersecting a point P' on the image plane. the radial height of P', h_i, is dependent on the image field angle ω_l. To the triangle $\Delta M_lO_lC_l$,

$$M_lC_l=\frac{\sin\beta_l}{\sin\omega_l}r_l \tag{7}$$

$$h_i=\tan\omega_l\left(\frac{r_l\sin\beta_l}{\sin\omega_l}-r_l+d_l\right) \tag{8}$$

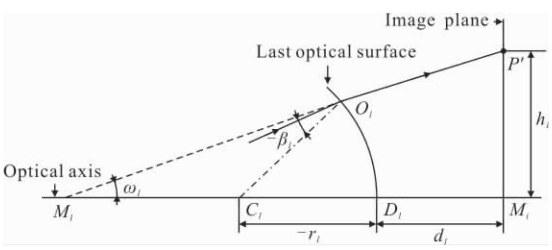


Fig.5 Optical path of the chief ray passing through the last optical surface and the intersecting point on the image plane

In Eqs.(7) and (8), r_l and β_l are the radius of curvature of the last optical surface and refraction angle of the chief ray at it; and d_l is the distance of the image plane from the last optical surface.

Applying Eqs.(6) and (8), the relation curve of h_i with respect to h_0 can be obtained with numerical calculation. We fit the curve with a proper function like

$$h_i=f(h_0) \tag{9}$$

Generally, it is a non-linear function, resulting from the non-similar imaging performance of a fisheye lens. Its higher order terms represent the image distortion.

1.3 Image at the projection plane

If we find the inverse function of expression (9),

$$h_0=f^{-1}(h_i) \tag{10}$$

It means that the projection of the scene at the projection plane can then be obtained in terms of the distorted image. As shown in Fig.1, for any point $P'(x',y')$ on the image plane $x'o'y'$, its polar coordinate will be

$$\begin{cases}h_i=\sqrt{x'^2+y'^2}\\ \theta_2=\arctan(y'/x')\end{cases} \tag{11}$$

With Eq. (10), the radial height h_0 of the corresponding point P'' at the projection plane $x''o''y''$ can be calculated; moreover, $\theta_1=\theta_2$. Thus the Cartesian coordinates of P'' is given below

$$\begin{cases}x''=h_0\sin\theta_2=\dfrac{h_0x'}{\sqrt{x'^2+y'^2}}\\ y''=h_0\cos\theta_2=\dfrac{h_0y'}{\sqrt{x'^2+y'^2}}\end{cases} \tag{12}$$

Equation (12) gives the corresponding relation between the pixel position at projection plane and the one at the image plane. Transformation of the pixel value from the latter to the former at the corresponding position will yield a picture without distortion.

2 Verification

2.1 Numerical simulation

In the following, we will use the method as described above to correct the image distortion of two fisheye lenses. The first one is Nikon 16 mm/F2.8 fisheye lens[9], every optical surface is identified with the corresponding number. Figure 6 shows the optical scheme of the fisheye lens. Its optical parameters are listed in Tab.1. We assume a plane grid object of size 480 mm×480 mm in the following numerical simulation, as shown in Fig.7. The object is placed at a distance of 90 mm from the lens and perpendicular to the optical axis of the fisheye lens.

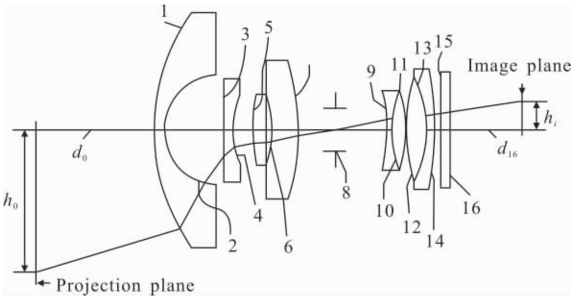


Fig.6 Optical scheme of Nikon 16 mm/F2.8 fisheye lens system

Tab.1 Parameters of Nikon 16 mm/*F*2.8 fisheye lens system

Surface i	Radius R_i/mm	Spacing d_i/mm	Index n_i/mm
0	∞	90.00	1.000 0
1	69.29	1.70	1.640 0
2	15.57	10.25	1.000 0
3	130.51	1.70	1.620 4
4	14.79	4.00	1.000 0
5	32.41	8.00	1.625 9
6	−15.30	9.00	1.796 7
7	−38.75	5.50	1.000 0
8(STO)	∞	5.50	1.000 0
9	−81.00	1.20	1.581 4
10	24.00	5.00	1.518 3
11	−20.61	0.10	1.000 0
12	46.73	5.50	1.518 6
13	−19.22	1.30	1.784 7
14	−40.71	0.88	1.000 0
15	∞	1.20	1.516 8
16	∞	38.95	1.000 0

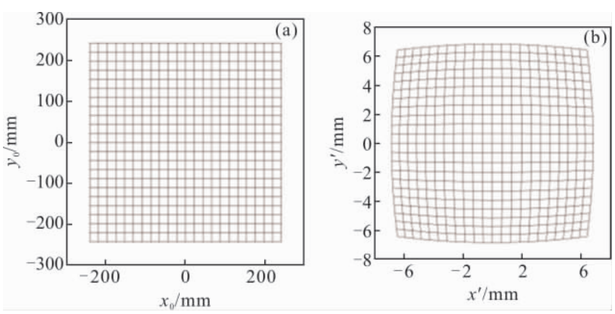


Fig.7 Image simulation of Nikon 16 mm/F2.8 fisheye lens: (a) object, (b) distorted image

The projection plane is assumed here to be coincident with the object plane; thus d_0=90 mm. From Eq.(6), as the field angle tends to 90°, h_0 tend to be infinite while the radial height of the ray on the image plane, h_i, is still limited by CCD or CMOS size of camera.We adopt the Fourier series to fit the numerical curve of Eq.(9). Compared to the polynomial fit, it has an advantage of better convergence of h_i as h_0 increases. Thus we assume,

$$h_i=\sum_{n=1}^{8} b_n\sin(nh_0\upsilon) \tag{13}$$

The coefficients of Eq.(13) by fitting are listed in Tab.2, and υ=0.001 6. The fit error is less than 0.01%.

Tab.2 Fit coefficients of Fourier series of Nikon 16 mm/*F*2.8 system

b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
16.323	0.439	−0.925	1.933	−1.184	0.621	−0.190	0.040

The inverse function of Eq.(13) obtained by use of the 'finverse (f)', an inverse function in the MATLAB program, gives the image on the projection plane. The fitting and inversing calculation will introduce small disagreement between object and image on projection plane. To measure this disagreement, we use the relative error of the radial height of the chief ray on the projection plane q:

$$q=\frac{\left|\sqrt{x''^2+y''^2}-\sqrt{x_0^2+y_0^2}\right|}{\sqrt{x_0^2+y_0^2}}\times 100\% \tag{14}$$

The calculation shows that the radial height error is within 0.3%, as shown in Fig.8, which is smaller than 1.54% as reported in Ref.[5].

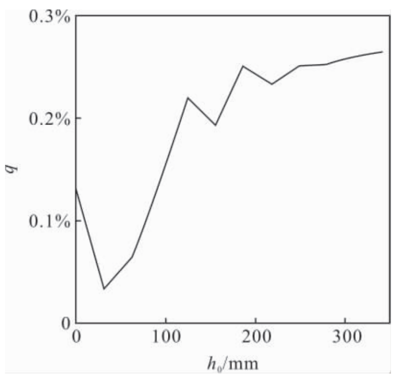


Fig.8 Relative error of the radial height between the object and the distortion-corrected image with Nikon 16 mm/F2.8 fisheye lens

The second example is a fisheye lens with field angle of 160° [10]. Figure 9 shows the optical scheme of the fisheye lens, every optical surface is identified with the corresponding number. Its optical parameters are listed in Table 3. We assume a plane grid object of size 1 000 mm×1 000 mm in the numerical simulation, as shown in Fig.10. It is placed at a distance of 100 mm from the lens and perpendicular to the optical axis of the fisheye lens. The coefficients of the Fourier series in Eq. (13) by means of fitting are listed in Tab.4, and υ=0.002 1. The calculation result shows that the relative error of the radial height is within 0.25%, as shown in Fig.11.

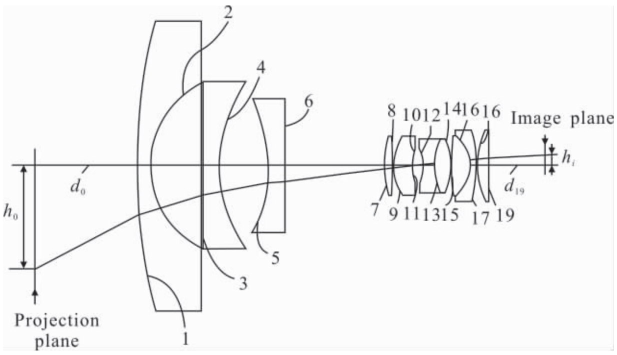


Fig.9 Optical schemes of fisheye lens with field angle of 160° system

Tab.3 Parameters of the second fisheye lens system

Surface i	Radius R_i/mm	Spacing d_i/mm	Index (n_i)
0	∞	100	1.000 0
1	544.99	13.56	1.516 8
2	92.22	53.09	1.000 0
3	∞	16.00	1.516 8
4	144.21	47.98	1.000 0
5	−144.21	16.00	1.516 8
6	∞	98.40	1.000 0
7	121.10	7.98	1.740 0
8	−403.26	0.71	1.000 0
9	53.97	18.16	1.740 0
10	49.62	2.87	1.000 0
11	∞	8.08	1.000 0
12	−46.68	11.13	1.688 9
13	60.33	16.46	1.651 6
14	−60.33	0.69	1.000 0
15	348.22	18.21	1.651 6
16	−36.57	5.97	1.740 0
17	−123.62	0.94	1.000 0
18	80.98	10.44	1.651 6
19	682.55	55.95	1.000 0

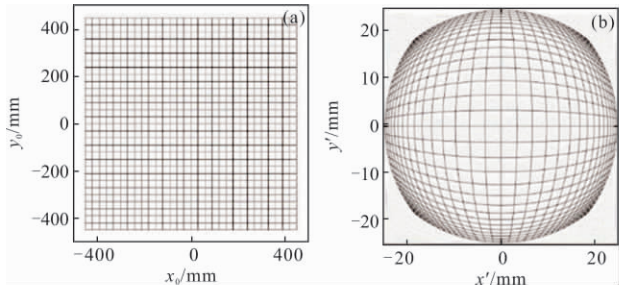


Fig.10 Image simulation of the fisheye lens with field angle of 1600: (a) object, (b) distorted image

Tab.4 Fit coefficients of Fourier series of the second fisheye lens system

b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
17.955	13.056	−2.770	2.577	−0.0550	−1.458	−3.437	−0.836

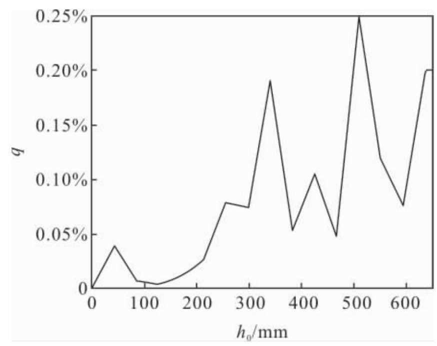


Fig.11 Relative error of the radial height between the object and the distortion-corrected image with the fisheye lens of Fig.9

2.2 Experiments

According to the discussions above, a program with the algorithm of image processing is developed in the MATLAB program. Its flow diagram is shown in Fig.12. As the projection plane is preferred to be positioned close to the object point, h_0 will be often so large that the pixel matrix to store the image on the projection plane exceeds computer memory. To produce a proper size of picture, we here introduce a scaling factor of k,

$$h_0'=kh_0 \tag{15}$$

Then according to Eq.(12), we have

$$\begin{cases} \overline{x''}=h_0'\sin\theta_2=\dfrac{h_0'x'}{\sqrt{x'^2+y'^2}} \\ \overline{y''}=h_0'\cos\theta_2=\dfrac{h_0'y'}{\sqrt{x'^2+y'^2}} \end{cases} \tag{16}$$

The pixel mapping from the image plane to the projection plane may not be one to one, especially in the area with serious distortion in the picture, affecting the image quality. We use the bilinear interpolation[11] to improve it.

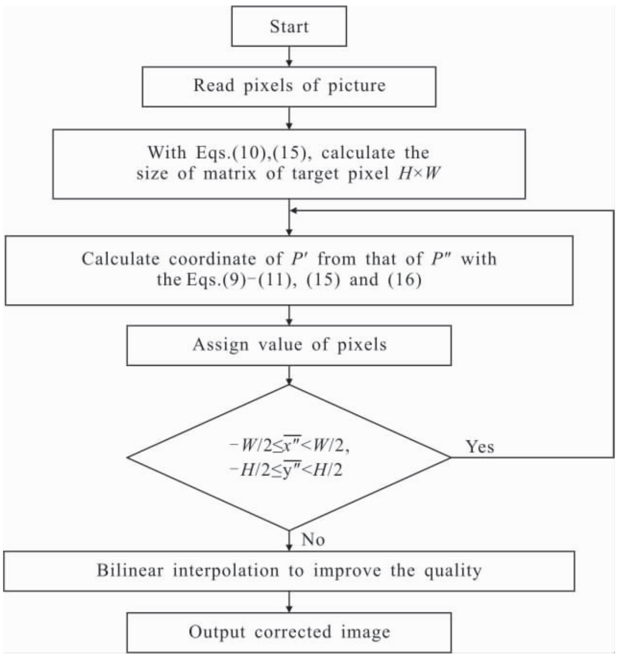
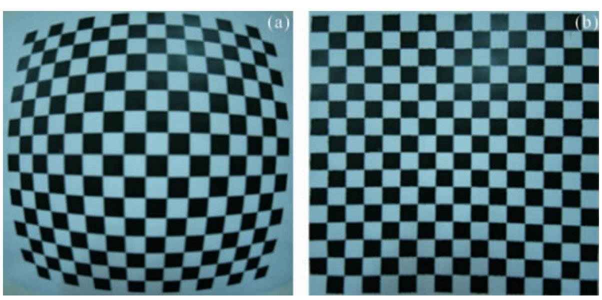


Fig.12 Flow diagram of the algorithm

Camera Nikon D810 and the Nikon 16 mm/F2.8 fisheye lens are used to perform two experiments. One is to use a plane grid object and the picture taken with the fisheye lens is shown as in Fig.13(a), and the distortion corrected image in Fig.13(b). Another one is to take a picture of an office scene with the lens, as shown in Fig.13(c), and the resultant image after processing is shown in Fig.13(d). The projection plane in the two experiments is assumed to be at a distance of 90 mm and of 1 500 mm from the lens, respectively.



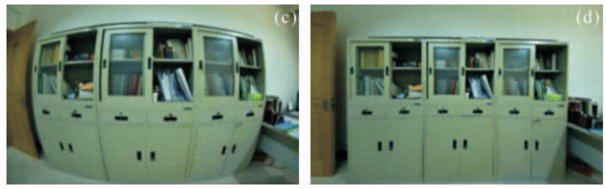


Fig.13 Two examples of the distortion-corrected image with Nikon 16 mm/F2.8 fisheye lens: (a) image of the plane grid; (b) distortion-corrected image of the plane grid; (c) image of an office scene; (d) distortion-corrected image of the office scene

3 Summary

From the above discussions, we can draw the following conclusions:

(1) From the numerical simulations and experiments of distortion correction of image, our method is feasible, and correction accuracy is quite satisfactory.

(2) Imaging model is relatively simple and universal; it only needs the optical parameters of fisheye lens, avoiding the complex process of camera calibration.

(3) The model discussed in the paper adopts a projection plane, and the restored image on the plane almost conforms to single-viewpoint projection; however, the range of the field angle is limited to be less than 180°.

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