### SNR of optical fiber current sensor reaching 0.2 accuracy class

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Abstract: Current ratio error and phase error, which are impacted by noise, are fundamental index of current sensor. Compared with conventional current sensor, optical fiber current sensor contains much more noise, thus has lower measurement accuracy for small signal. This has restricting practical use for optical fiber current sensor. Statistical characteristic of current ratio error and phase error under white Gaussian noise was analyzed, so was the standard of signal to noise ratio (SNR) when it can meet 0.2 accuracy class. Furthermore, formula of SNR was deduced and influencing factors of SNR was discussed. Theoretical analysis and experiments indicate that the SNR of optical fiber needs to be 29 dB to meet the demand of 0.2 accuracy class. When the rated current and update rate are confirmed, only reducing noise and increasing the number of loops of optical fiber current sensor can improve SNR.

Key words: optical fiber current sensor; SNR; 0.2 accuracy class; current ratio error; phase error CLC number: TM937 Document code: A Article ID: 1007-2276(2014)11-3704-05

## 满足 0.2 级的光纤电流互感器信噪比分析

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摘 要: 电流比值误差和相位误差是检验互感器的重要指标,噪声对其有较大影响。光纤电流互感器 不同于传统电流互感器的重要特征之一即是存在较大噪声,在测量小电流时表现尤为显著,已经成为 阻碍光纤电流互感器实用化进程的重要难题。分析了高斯白噪声背景下电流比值误差和电流相位误 差的统计特征,研究了 0.2S 级下电流互感器数字输出信号信噪比所需满足的条件,探讨了电流互感 器信噪比的计算方法以及影响信噪比的因素,并设计实验验证了理论分析的正确性。分析表明,为满 足 0.2S 级光纤电流互感器信噪比需达到 136,在额定电流以及数据更新率一定的情况下,仅有通过降 低噪声以及增加敏感线圈缠绕圈数的方法可以提高系统信噪比。

关键词:光纤电流互感器; 信噪比; 0.2级; 比值误差; 相位误差

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In-line Sagnac interferometer current sensor is a kind of novel current sensor which possess high measurement accuracy, wide frequency response range and digitized output, therefore, it gradually become key primary equipment of smart power grids. According to performance test for electronic current sensor conducted by State Grid Corporation of China in 2011<sup>[1]</sup>, measuring error of in-line Sagnac interferometer current sensor were seriously over 0.2 accuracy class when testing small signal. It is mainly reflected in wide fluctuation of both current ratio error and phase error. This is the principle shortcoming of optic fiber current sensor. Thus, the study on signal to noise ratio (SNR) of optical fiber current sensor which can meet the standard of 0.2 accuracy class has great significance.

Calibrators tend to adopt Fourier transform to measure current ratio error and phase error and noise has negative influence on these results<sup>[2]</sup>. For major noise of optical fiber current sensor is white noise<sup>[3]</sup>, it is necessary to discuss the statistical characteristic of current ratio error and phase error under white Gaussian noise. Linear birefringence introduced by sensing head will lead to current ratio error, and it is the major problem of error on scale factor along with temperature<sup>[4]</sup>. Consequently, the length of sensing fiber need to be reduced under the condition that the measure accuracy is fulfilled to improve reliability of optical fiber current sensor in temperature test. However, the length of sensing fiber is determined only by experience. At present, there is no relative study on it. Aiming at above problems, statistical characteristic of current ratio error and phase error under white Gaussian noise is analyzed, SNR to meet 0.2 accuracy class is calculated, formula of SNR is deduced and influencing factors of SNR is discussed. The paper has guiding significance on noise research and engineering application of optical fiber current sensor.

# 1 Statistical characteristic of current ratio error and phase error under white Gaussian noise

Noise of optical fiber current sensor mainly comes from photo signal, interference and signal detection <sup>[5]</sup>. Because of working principle and composition of optical fiber current sensor, its noise is considerable, which will result in error on current ratio error and phase error. For major noise of optical fiber current sensor is white noise, so statistical theory on white Gaussian noise can be applied to study the feature of current ratio error and phase error<sup>[6]</sup>. White Gaussian noise means the density function of noise satisfy Gaussian distribution and power spectral density of noise is a constant. We assume the current to be measured is sinusoidal signal, so the output of optical fiber current sensor can be expressed as:

$$s(t) = x(t) + z(t)$$
 (1)

where z(t) is white noise whose variance is  $\delta^2$ ; x(t) is sinusoidal signal whose frequency is  $f_0$ ; amplitude is A and initial phase is  $\theta$ . After adding window function whose length is N to s(t), sample sequence can be acquired as:

$$s_{\omega}(n) = x(n) \omega_{N}(n) + z(n) \omega_{N}(n)$$
(2)

After discrete Fourier transformation:

$$\vec{S}_{\omega}(\mathbf{k}) = \vec{X}_{\omega}(\mathbf{k}) + \vec{Z}_{\omega}(\mathbf{k}), \ \mathbf{k} = 0, 1, \cdots, N/2$$
(3)

where  $\vec{S}_{\omega}(k)$ ,  $\vec{X}_{\omega}(k)$  and  $\vec{Z}_{\omega}(k)$  respectively are frequency of  $s_{\omega}(n)$ ,  $x(n)\omega_N(n)$  and  $z(n)\omega_N(n)$ . So

$$S_{\omega}(k_{r}^{*}) = \sqrt{[X_{\omega}^{R}(k_{r}^{*}) + Z_{\omega}^{R}(k)]^{2} + [X_{\omega}^{I}(k_{r}^{*}) + Z_{\omega}^{I}(k)]^{2}}$$
(4)

$$\mathbf{S}_{\omega}^{\mathsf{P}}(\mathbf{k}_{r}^{\star}) = \arctan\left(\frac{\mathbf{X}_{\omega}(\mathbf{k}_{r}^{\star}) + \mathbf{Z}_{\omega}(\mathbf{k})}{\mathbf{X}_{\omega}^{\mathsf{R}}(\mathbf{k}_{r}^{\star}) + \mathbf{Z}_{\omega}^{\mathsf{R}}(\mathbf{k})}\right)$$
(5)

where superscript **R** represents real part and superscript **I** represents imaginary part. Eq.(4) and Eq.(5) can be expressed as Eq. (6) and Eq. (7) after using Taylor formula:

$$S_{\omega}(\mathbf{k}_{r}^{\star}) \approx X_{\omega}(\mathbf{k}_{r}^{\star}) + ZA_{\omega}(\mathbf{k}_{r}^{\star})$$
(6)

$$\mathbf{S}_{\omega}^{P}(\mathbf{k}_{r}^{\star}) \approx \mathbf{X}_{\omega}^{P}(\mathbf{k}_{r}^{\star}) + \mathbf{ZP}_{\omega}(\mathbf{k}_{r}^{\star})$$
(7)

where

$$ZA_{\omega}(\mathbf{k}_{r}^{*}) = \frac{\mathbf{X}_{\omega}^{R}(\mathbf{k}_{r}^{*})}{\mathbf{X}_{\omega}(\mathbf{k}_{r}^{*})} \mathbf{Z}_{\omega}^{R}(\mathbf{k}_{r}^{*}) + \frac{\mathbf{X}_{\omega}^{I}(\mathbf{k}_{r}^{*})}{\mathbf{X}_{\omega}(\mathbf{k}_{r}^{*})} \mathbf{Z}_{\omega}^{I}(\mathbf{k}_{r}^{*}) = \cos(\mathbf{X}_{\omega}^{P}(\mathbf{k}_{r}^{*})) \mathbf{Z}_{\omega}^{R}(\mathbf{k}_{r}^{*}) + \sin(\mathbf{X}_{\omega}^{P}(\mathbf{k}_{r}^{*})) \mathbf{Z}_{\omega}^{I}(\mathbf{k}_{r}^{*})$$
(8)

$$ZP_{\omega}(\mathbf{k}_{r}^{*}) = \frac{\mathbf{X}_{\omega}(\mathbf{k}_{r}^{*})}{\mathbf{X}_{\omega}^{2}(\mathbf{k}_{r}^{*})} \mathbf{Z}_{\omega}^{1}(\mathbf{k}_{r}^{*}) - \frac{\mathbf{X}_{\omega}(\mathbf{k}_{r}^{*})}{\mathbf{X}_{\omega}^{2}(\mathbf{k}_{r}^{*})} \mathbf{Z}_{\omega}^{R}(\mathbf{k}_{r}^{*}) = \frac{\cos(\mathbf{X}_{\omega}^{P}(\mathbf{k}_{r}^{*}))\mathbf{Z}_{\omega}^{1}(\mathbf{k}_{r}^{*}) - \sin(\mathbf{X}_{\omega}^{P}(\mathbf{k}_{r}^{*}))\mathbf{Z}_{\omega}^{R}(\mathbf{k}_{r}^{*})}{\mathbf{X}_{\omega}(\mathbf{k}_{r}^{*})}$$
(9)

According to statistical characteristic of white Gaussian noise, Eq. (10) and Eq. (11) can be acquired:

$$S_{\omega}(\mathbf{k}_{r}^{*}) \rightarrow \mathbf{N}\left(\mathbf{X}_{\omega}(\mathbf{k}_{r}^{*}), \frac{\sigma^{2}\mathbf{P}\omega}{2\mathbf{N}}\right)$$
 (10)

$$\mathbf{S}_{\omega}^{\mathsf{P}}(\mathbf{k}_{r}^{*}) \rightarrow \mathbf{N}\left(\mathbf{X}_{\omega}^{\mathsf{P}}(\mathbf{k}_{r}^{*}), \frac{\sigma^{2}\mathbf{P}\omega}{2\mathbf{N}\mathbf{X}_{\omega}^{2}(\mathbf{k}_{r}^{*})}\right)$$
(11)

Thus, current ratio error (AER) and phase error (AER) can be expressed as:

$$AER = \frac{S_{\omega}(k_{r}^{\star})}{X_{\omega}(k_{r}^{\star})} - 1 \longrightarrow N \left( 0, \frac{\sigma^{2}P\omega}{2NX_{\omega}^{2}(k_{r}^{\star})} \right)$$
(12)

$$\mathsf{PER} = \mathsf{S}_{\omega}^{\mathsf{P}}(\mathbf{k}_{\mathsf{r}}^{\star}) - \mathsf{X}_{\omega}^{\mathsf{P}}(\mathbf{k}_{\mathsf{r}}^{\star}) \longrightarrow \mathsf{N}\left(0, \frac{\sigma^{2}\mathsf{P}\omega}{2\mathsf{N}\mathsf{X}_{\omega}^{2}(\mathbf{k}_{\mathsf{r}}^{\star})}\right) \quad (13)$$

Considering  $X_{\omega}(k_r^*) = \frac{A}{2} (\nabla f_r^1)$ , and SNR is indicated as SNR= $A^2/\sigma_r^2$ , so

$$\mathsf{AER} \to \mathsf{N}\left(0, \frac{2}{\mathsf{N} \cdot \mathsf{SNR}}, \frac{\mathsf{P}\omega}{\mathsf{W}^2(\nabla \mathsf{f}_r^1)}\right) \tag{14}$$

$$\mathsf{PER} \rightarrow \mathsf{N} \left( 0, \frac{2}{\mathsf{N} \cdot \mathsf{SNR}} \frac{\mathsf{P}\omega}{\mathsf{W}^2(\nabla \mathsf{f}_r^1)} \right) \tag{15}$$

Accordingly, current ratio error and phase error have direct relationship with the length of window function and SNR. According to 0.2 accuracy class, current ratio error should be within 0.75 and phase error should be within 30' when measuring 5% of rated current. Consequently, 0.2 accuracy class has higher demand on phase error than current ratio error. If phase error can meet the demand, then the current ratio error must also meet the demand considering phase error and current ratio error has similar distribution. So phase error is applied as the study object.

According to mathematical statistics, 1 -  $\alpha$ 

confidence intervals of phase error is:

$$(-\mu_{1-\alpha/2} \left[ \frac{2\mathsf{P}\omega}{\mathsf{N}\cdot\mathsf{SNR}} \right]^{\frac{1}{2}} \frac{1}{|\mathsf{W}(\nabla \mathsf{f}_{\mathsf{r}}^{1})|},$$
$$\mu_{1-\alpha/2} \left[ \frac{2\mathsf{P}\omega}{\mathsf{N}\cdot\mathsf{SNR}} \right]^{\frac{1}{2}} \frac{1}{|\mathsf{W}(\nabla \mathsf{f}_{\mathsf{r}}^{1})|}$$
(16)

where  $\mu_{1 - \alpha/2}$  is  $1 - \alpha/2$  quantile of standard normal distribution U ~N (0,1). Assuming PER<sub>v</sub> is accepted error threshold, then

$$\mu_{1-\alpha/2} \left[ \frac{2\mathsf{P}\omega}{\mathsf{N}\cdot\mathsf{SNR}} \right]^{\frac{1}{2}} \frac{1}{|\mathsf{W}(\nabla \mathsf{f}_{\mathsf{r}}^{1})|} \leq \mathsf{PER}_{\mathsf{v}} \qquad (17)$$

Assume  $\alpha = 0.01$ , then  $\mu_{1-\alpha/2} = 2.58$ , PER<sub>v</sub> =  $\pi/360$ , with rectangular window, P $\omega = 1$ ,  $|W(\nabla f_r^1)| = 0.5271$ , N=800, so SNR=29 dB. Therefore, only when SNR of the optical fiber current sensor is up to 29 dB can phase error of the sensor is within 30' under most condition.

#### 2 SNR of optical fiber current sensor

In-line Sagnac interferometer current sensor uses Faraday Effect and Ampere Law to measure current. Configuration of in-line Sagnac interferometer current sensor is shown in Fig.1.

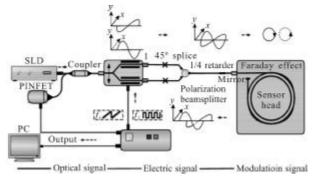


Fig.1 Configuration of in-line Sagnac interferometer current sensor

The base of Y -junction of multifunctional integrated optic chip (MIOC) is connected to the coupler while the arms of the Y -junction are interfaced with the polarization beam splitter. Optical signals input to the MIOC divide at the Y-junction to form optical signals which are transverse electric (TE) mode and transverse magnetic (TM) mode. The

orthogonal waves are converted into left and right circular waves at the 1/4 retarder which is 45 degrees countershaft wielding with polarization maintaining fiber. With magnetic field generated by electric current, two circular waves will have phase difference which is proportional with current to be measured. After the mirror, two circular waves return along the same route and its phase difference doubles. After passing through the 1/4 retarder, orthogonal waves will have direction interchange (the TE mode will become TM mode and the TM mode will become TE mode). The returning orthogonal waves propagate in the upper and lower branches of the MIOC in reciprocal paths, and are brought to interference. The digital closed-loop detection circuit detects the signal. In order to improve measurement accuracy, the digital closed-loop detection circuit processes the signal with square wave modulation and closed loop detection<sup>[7]</sup>. The output expression is:

 $I_D = K_{loss} \cdot I_0 \cdot [1 \mp \sin(\theta_F + \varphi_f)]$  (18) where  $\theta_F = 4N_i VI$ ;  $K_{loss}$  is the loss in the optical path;  $I_0$ is light intensity of the SLD.  $N_i$  is the loops of sensing fiber, V is Verdet constant and I is the current magnitude.

The feedback  $\varphi_{f}$  is achieved by digital phase ramp. The digital output of the system is the step height of digital phase ramp which is represent as  $D_{out}$  (t), therefore the analog step height is:

$$\Delta V_{f}(t) = V(t) - V(t - \tau) = K_{DA} \cdot D_{out}(t)$$
(19)

where  $K_{DA}$  is conversion coefficient of digital output and analog output,  $K_{DA} = V_{pp}/2^{N_1}$ ;  $V_{pp}$  is the peak-to-peak value of analog phase ramp and  $N_1$  is bit of Digitalto-Analog Converter. Assuming  $K_{fp}$  is modulation coefficient of MIOC, then the feedback  $\varphi_f$  is:

$$\varphi_{f} = \mathbf{K}_{f_{p}} \cdot \frac{\mathbf{V}_{PP}}{2} \cdot \mathbf{D}_{out}(t) = -4\mathbf{NVI}$$
(20)

then

$$D_{\text{out}}(t) = -\theta_{\text{F}}(t) \cdot \frac{2^{N_1}}{K_{\text{fp}} \cdot V_{\text{PP}}} = -4NV \cdot \frac{2^{N_1}}{K_{\text{fp}} \cdot V_{\text{PP}}} \cdot I \qquad (21)$$

Considerin g system output will accumulate M times before data collection. Then the digital output

S<sub>out</sub> is:

$$S_{\alpha t}(t) = 4MNV \cdot \frac{2^{N_1}}{K_{fp} \cdot V_{pp}} \cdot I = KI$$
(22)

Where the scale factor K is:

$$\mathbf{K} = -4\mathbf{MNV} \cdot \frac{2^{N_1}}{K_{\rm fp} \cdot V_{\rm PP}}$$
(23)

According to the national standard, the output of current sensor within rated bandwidth when there is no primary current act on the current sensor is noise; while signal within rated bandwidth under primary current with 50 Hz frequency is useful signal. Assuming that the noise is white noise whose variance is  $\delta^2$ , then the SNR of current sensor can be presented as:

$$SNR = \frac{D[S_{out}^2]}{\sigma^2} = K^2 \frac{D[I^2]}{\sigma^2}$$
(24)

On the basis of Equation (24), the method to improve SNR is to reduce noise, enhance current to be measured and add the loops of sensing fiber for M is a constant value.

#### 3 Experimental results

To verify the feasibility of above deduction, we designed the experimental setup as Fig.2.

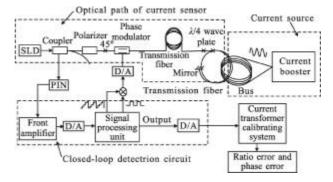


Fig.2 Schematic diagram of experimental setup

The experimental setup consists of signal current booster, optical fiber current sensor and current transformer calibrating system. The current booster generates sinusoidal current with fixed frequency of 50 Hz to drive optical path of the current sensor. Under Faraday effect, phase of light propagating in the sensing fiber will change. The output of closed-loop detection circuit will enter the test channel of current transformer calibration system at 400 kHz. Ratio error and phase error will then be acquired.

Firstly, experiment is conducted with the optical fiber current sensor whose SNR is 28 dB and the ratio error and phase error is shown in shown in Fig.3. Signal in dark line is ratio error and signal in light line is phase error.

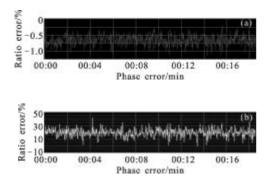


Fig.3 Ratio Error and phase error when SNR is 28 dB

Then optical fiber current sensor whose SNR is 29 dB is tested. The performance data under room temperature is shown in Fig.4.

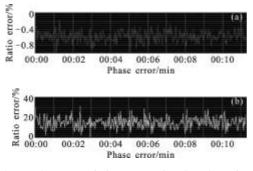


Fig.4 Ratio Error and phase error when SNR is 20 dB

It is easy to find that optical fiber current sensor whose SNR is 29 dB improve its accuracy to meet the 0.2 class while optical fiber current sensor whose SNR is 28 dB can not reach the standard.

#### 4 Conclusions

This paper analyzes statistical characteristic of ratio error and phase error under white Gaussian noise, as well as calculate the SNR which can meet 0.2 accuracy class. In addition, formula of SNR is deduced and influencing factors of SNR is discussed. Theoretical analysis and experiments indicate that the SNR of optical fiber need to be 29 dB to meet the demand of 0.2 accuracy class. When the rated current and update rate is confirmed, only reducing noise and increasing the number of loops of optical fiber current sensor can improve SNR.

#### References:

- [1] EPRI of China, Performance test results of electronic current transformers [EB/OL]. [2012-02-22]. <u>http://www.doc88</u>. com/p-956282641622.html.
- [2] AQSIQ, SAC. GB/T 20840. 8 2007. Instrument transformerspart 8: Electronic current transformers [S]. Beijing: Standards Press of China, 2007. (in Chinese)
- [3] Lefvre H C. The Fiber Optic Gyroscope [M]. Beijing: National Defense Industry Press, 2002. (in Chinese)
- [4] Zhang Chunxi, Li Chuansheng, Wang Xiaxiao, et al. Design principle for sensing coil of fiber-optic current sensor based on geometric rotation effect [J]. Applied Optics, 2012, 51 (18): 3977-3988.
- [5] Zhang Weixu. The Fiber-Optic Gyroscope and Its Application
   [M]. Beijing: National Defense Industry Press, 2008. (in Chinese)
- [6] Ding Kang, Xie Ming, Yan Zhijian. The Theory and Technology of Discrete Spectrum Correction [M]. Beijing: Science Press, 2008. (in Chinese)
- [7] Gao Jinzhan. Detection of Weak Signals [M]. Beijing: Tsinghua University Press, 2011. (in Chinese)