Principle, applications, evaluation and development of time lens

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Abstract: Time lens is based upon space-time duality and has been contributed much attention during the last decade as a widely used optical instrumentation. Improvement of time lens is always enhanced by development of photonics as both engineering requirements and theoretical driving. A historical overview of how this powerful framework had been exploited to develop ultra-fast optical instruments was presented. Current state of implementing time lens by phase modulator (PM), sum-frequency generation (SFG), cross-phase modulation (XPM) and four-wave mixing (FWM) were summarized and analyzed by mathematic description. Then, limitations of different implementations of time lens for applications above were analyzed, accordingly. In addition, pulse magnification and time to frequency conversion as the main applications for ultra-fast pulse measurement by time lens were outlined with emphasizing on the evaluation by performances including resolution and record length. Furthermore, some ultra-fast nonlinear principle including surface-plasmon enhanced ultra-fast second- and third-order optical nonlinearities in metallic nanostructure, strong third-order optical nonlinearity induced high efficient FWM in graphene as potential theoretical and technological opportunities to improve time lens were presented and discussed.

Key words: time lens; space-time duality; temporal imaging; ultra-fast nonlinear optics; ultra-fast optical signal processing

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时间透镜的原理、应用以及性能和发展

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摘 要: 时间透镜基于时空二元性原理,近年来得到快速发展与广泛应用。时间透镜的发展源自光电子学中工程技术需求和理论发展的双重动力。给出了时间透镜作为超快光学仪器发展历程的综述。对相位调制器、和频产生、交叉相位调制以及四波混频等当前时间透镜的主要实现方案的原理和性能进

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行了分析和数学描述。相应地,分析了不同实现方案的原理限制以及应用中面临的问题。接下来,将脉冲放大和时频域转换用于超快脉冲检测作为最有代表性的应用进行了说明,其中,对最重要的技术指标分辨率和记录长度进行了定量分析。最后,对一些超快非线性光学原理,如金属纳米结构中表面等离子体激元增强的二阶和三阶光学非线性、石墨烯中强三阶光学非线性导致的四波混频,作为时间透镜发展的潜在机会进行了理论和技术讨论。

关键词: 时间透镜; 时空二元性; 时间成像; 超快非线性光学; 超快光信号处理

0 Introduction

Based upon space-time duality[1-2], time lens has been exploited over the last decade to provide widely employment in photonic fields of pulse magnification and generation, time to frequency conversion, optical packet and pulse compression and pulse shaping[3-7].

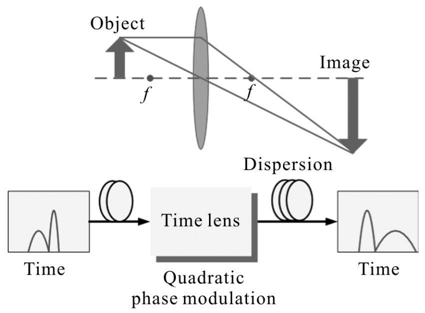
A time lens imposes a temporal quadratic phase modulation onto the incident light, analogous to a spatial lens imposing a spatial quadratic phase onto the wave-front. In practice, the quadratic phase modulation can be achieved approximately by applying a sinusoidal radio frequency (RF) signal to an electro-optic phase modulator [8-10] or employing some nonlinear processes including sum- & difference-frequency generation (SFG, DFG)[11], self- & cross-phase modulation (SPM, XPM)[12] and four-wave mixing (FWM)[13-14]. With proper dispersion, a temporal imaging system can be realized by time lens to achieved pulse magnification, compression and time to frequency conversion.

The aim of this paper is to show how time lens can be exploited to develop practical instrumentation for ultra-fast optical signal processing by triggering numerous literatures from the early 1990s to the present.

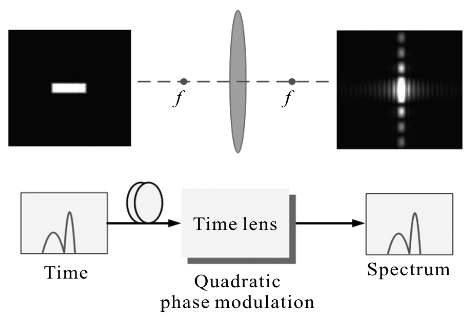
1 Principle

1.1 Space-time duality

Time lens relies on the principles of the analogy between paraxial diffraction and narrowband dispersion, namely, space-time duality (shown in Fig.1). Due to this duality, many spatial imaging systems have the corresponding temporal analogues. In Fig.1 (a), a magnified image is obtained by a spatial imaging system, a optical pulse can be compressed or decompressed by a temporal imaging system with time lens, analogously. And in Fig.1 (b), space lens can perform the Fourier transform of an object positioned on focal plane of the lens. Likewise, a time lens (precisely speaking, a thin lens here) can perform the Fourier transform of an input pulse after propagation through a dispersive focal length.



(a) Temporal imaging by time lens for pulse magnification



(b) Fourier transform by time lens for time to frequency conversion

Fig.1 Schematic illustration of space to time duality: time lens and analogy to a space lens

In order to avoid resolving Maxwell's equations directly, the theory of space-time duality can be constructed by two approximations to the wave equation [1-2]. For the spatial problem, monochromatic and paraxial approximations are assumed, and complementary set of approximations can be invoked for the temporal problem. In order to describe the pulses in general, it is necessary to limit the temporal-frequency spectrum to a suitable range, then the propagation of any spectral component within the wave can be accounted for by a Taylor-series expansion of the propagation constant $\beta(\omega)$ to second order in ω. Then, the spatial profile of the wave is ignored and approximated as an infinite plane wave. These two approximations simplify the wave equation and result in a parabolic differential equation for the space-time evolution of the pulse envelope. The paraxial wave equation can be expressed as

$$E_z=-\frac{i}{2k}(E_{xx}+E_{yy}) \tag{1}$$

where E denotes field amplitude, the subscripts on the field amplitudes imply partial differentiation with respect to that coordinate, and k is the wave-number.

Analogously, an optical pulse propagating in the z-direction with a slowly varying envelope function $A(\xi,\tau)$ can be written in the governing partial differential equation for narrowband dispersion:

$$\frac{\partial A(\xi,\tau)}{\partial\xi}=\frac{i}{2}\frac{d^2\beta}{2d\omega^2}\frac{\partial^2 A(\xi,\tau)}{\partial\tau^2} \tag{2}$$

where $\tau=(t-t_0)-\left(\frac{z-z_0}{v_g}\right)$, $\xi=z-z_0$, t_0 and z_0 are arbitrary references and v_g is the group velocity. Eqs.(1) and (2) present the analogical mathematic description of paraxial diffraction and narrowband dispersion, which is the foundation of the theoretical analyzing later.

1.2 Time lens

The relevant spatial-phase function for paraxial diffraction is:

$$\varphi(x,y)=\frac{k(x^2+z^2)}{2f_s} \tag{3}$$

where f_s is the focal length. It is found from Eq.(3) that a space lens produces a quadratic phase modulation on the profile variables (x,y), and it suggests that in the time domain, a time lens can produce a quadratic phase modulation on the local time variable τ. So that the phase function of a time lens could be described as $\varphi(\tau)=\mu\tau^2$, where μ is a constant. By keeping with the spirit of the space-time duality, $\varphi(\tau)$ can be given by the following expression:

$$\varphi(\tau)=\frac{\omega_0\tau^2}{2f_T} \tag{4}$$

where f_T is the focal time of time lens, and ω_0 is the optical carrier frequency. Then, a general expression for the focal time f_T of a time lens realized by any physical process can be obtained by comparing Eq.(4) to a Taylor-series expansion:

$$\varphi(\tau)=\varphi_0(\tau_0)+(\tau-\tau_0)\frac{d\varphi}{d\tau}+\frac{(\tau-\tau_0)^2}{2!}\frac{d^2\varphi}{d\tau^2}+\cdots \tag{5}$$

Equating the second-order term (with $\tau_0=0$) to Eq.(4), we get

$$f_T=\frac{\omega_0}{d^2\varphi/d\tau^2} \tag{6}$$

Phase function (4) and the focal time Eq.(6) are two elementary parameters for design and realizing a time lens. It is worth to noting that space-time duality still presents some opportunity to invite some useful techniques except time lens (accurately speaking, thin time lens), such as time prism[2] and graded-index time lens[8]. It is a meaningful work to extend the space-time duality in this way.

Next, tempo ral imaging and time to frequency conversion by time lens are analyzed as two essential functions, based on which some further applications in pulse magnification and compression, pulse generation and shaping, ultra-fast pulse measurement can be achieved.

1.3 Temporal imaging

In spatial domain, a light beam can be focused to a spot by a space lens with suitable distance for diffraction, which is analogous to pulse compression in the time domain. A temporal imaging system can be obtained by a time lens with suitably chosen pre- and post-dispersion with respect to the characteristics of the time lens, as shown in Fig.2. Then, the output waveform from the temporal imaging system is a scaled replica of the input waveform; stretched or compressed in time with a concomitant scaling in power.

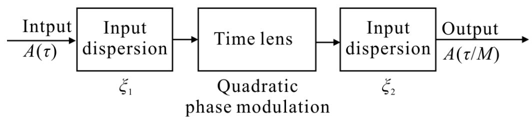


Fig.2 Schematic illustration of a general temporal imaging system

There is a condition under which the output waveform could reaso nably be called an "image" of the input waveform. This would be satisfied if the output waveform has the same envelope profile, but perhaps with an altered time scale. Just similar to the spatial imaging condition, this temporal imaging condition can be written as:

$$\frac{1}{\xi_1 \frac{d^2\beta_1}{d\omega_1^2}}+\frac{1}{\xi_2 \frac{d^2\beta_2}{d\omega_2^2}}=-\frac{\omega_0}{f_T} \tag{7}$$

where $\xi_{1,2}$ are input and output propagation distance, respectively. When this condition is satisfied, the principal result of temporal imaging can be expressed as:

$$A(\xi_2,\tau)=\frac{1}{2\pi\sqrt{M}}\exp\left[\frac{i\omega_0\tau^2}{2Mf_T}\right]\int_{-\infty}^{\infty}\tilde{A}(0,\omega)\exp\left[i\omega\frac{\tau}{M}\right]d\omega=\frac{1}{\sqrt{M}}\exp\left[\frac{i\omega_0\tau^2}{2Mf_T}\right]A\left(0,\frac{\tau}{M}\right) \tag{8}$$

where $A(\xi_2,\tau)$ is the output envelope, $\tilde{A}(0,\omega)$ is the frequency spectrum of the input waveform envelope $A(\xi,\tau)$. M is the magnification factor, by which the time scale is altered and given by the ratio of the output dispersion to the input dispersion:

$$M=-\frac{\xi_2 \frac{d^2\beta_2}{d\omega_2^2}}{\xi_1 \frac{d^2\beta_1}{d\omega_1^2}} \tag{9}$$

where minus sign of M means a temporal reversal.

It is described in Eq.(8) and (9) that the output waveform $A(\xi_2,\tau)$ is seen to be a replica of the input waveform scaled in time with $1/M$. When $|M|>1$, the temporal imaging system is used for pulse magnification, and $|M|<1$ is for compression. In addition, there is a quadratic phase term and an amplitude scaling $1/\sqrt{M}$ as that in the spatial case.

For spatial imaging, resolution is ultimately governed by the speed of the lens. In fact, a practical choice of resolution depends on the application. However, a general and useful definition is to consider two pulses "resolved" when they are separated at the output by the width of the impulse response of system. By approximating the object distance as equal to the focal length, the input resolution of the temporal imaging system can be written as:

$$\delta\tau=T_0\frac{f_T}{\tau_a}=T_0 f_T^{\#} \tag{10}$$

where $T_0=2\pi/\omega_0$ is the period of the optical carrier, and τ_a is the time aperture (also called aperture size) of time lens, which is essentially determined by the implementations of time lens. In addition, a f-number can be defined as the ratio of focal time to the time aperture $f_T^{\#}=f_T/\tau_a$.

For a time lens and temporal imaging system, focal time, f-number, magnification factor and the resolution are generally considered as the elementary but essential performances. Further performances for applications of time lens are mainly determined by these essential performances.

1.4 Time to frequency conversion

For a spa ce lens, an object at the front focal plane will produce a Fourier transform of the object at the back focal plane, shown in Fig.1(b). Analogously, the Fourier transform of a temporal waveform of pulse is its optical spectrum by a time lens, which is termed time to frequency conversion[3,7].

In the temporal imaging system, if the amount of dispersion is adjusted such that the input pulse $A_1(\tau)$ can be operated at the front focal plane. In this case, output pulse $A_2(\tau)$ is a Fourier transform of $A_1(\tau)$ multiplied by a quadratic phase factor and the Fourier transform of $A_2(\tau)$ can be measured directly by a optical spectrum analyzer (OSA) and expressed as:

$$\tilde{A}(\omega-\omega_0)=A_1(t),\ t=(\omega-\omega_0)f_T/\omega_0 \tag{11}$$

Therefore, input waveform can be found by measuring the spectrum of output pulse and scaling the wavelength (λ) axis to a time (t) axis by

$$t=(\omega-\omega_0)f_T/\omega_0=(\lambda-\lambda_0)f_T/\lambda \tag{12}$$

For time to frequency conversion, except resolution, the record length is generally utilized to describe the available longest input pulse.

2 State-of-the-art implemntations of time lens

A time lens can be more specifically defined as a physical mechanism that imparts a quadratic phase modulation or a linearly chirp on an input optical pulse. Currently, there are several solutions for implementing a time lens: FM, SPM, XPM and parametric process including three-wave mixing (SFG and DFG) and FWM. Different implementations of time lens are illustrated in Tab.1 and described in detail as follows.

2.1 Phase modulator

In the earlier research works, time lenses are firstly implemented with electro-optic phase modulator [8-9]. When a sinusoidal RF signal is applied to a phase modulator, the quadratic phase modulation can be provided approximatively.

Tab.1 Parameters for implementations of time lens

Linear process		Nonlinear processes		
		XPM	Parametric process	
Parameters	Phase modulator		SFG	FWM
$\varphi(\tau)$	$\frac{\pi V_0}{V_\pi}\left(1-\frac{\omega_m^2\tau^2}{2}\right)$	$-2l_0\gamma_0\lvert A_p(\tau)\rvert^2$	$-\frac{\tau^2}{2\xi_f\beta''}$	$\frac{\tau^2}{\beta_2^{(p)}L_p}$
f_T	$\frac{\omega_0 V_\pi}{\pi\omega_m^2 V}$	$\frac{\omega_0}{4l_0\gamma_0 a}$	$-\omega_0\xi_f\beta''$	$\omega_0\beta_2^{(p)}L_s$
$f_T^{\#}$	$\frac{\omega_0 V_\pi}{\pi\omega_m V}$	$\frac{\omega_0}{4\tau_a l_0\gamma_0 a}$	$\omega_2/\Delta\omega_p$	$\tau_{pump}/\sqrt{2}\,T_0$
τ_a	$\tau_a=\frac{1}{2\pi\omega_m}$	τ_p	$-\Delta\omega_p\omega_0\xi_f\beta''/\omega_2$	//
$\delta\tau$	$\frac{1}{\Gamma_0 f_m}$	$\frac{\pi}{\tau_p 2l_0\gamma_0 a}$	$\Delta\tau_{p0},\Delta\tau_{p0}/\lvert M\rvert$	$\tau_{pump}/\sqrt{2}$
M	$-\xi_2\frac{d^2\beta_2}{d\omega_2^2}\Big/\xi_1\frac{d^2\beta_1}{d\omega_1^2}$		$M=-\varphi_2''/\varphi_1''$	$-\xi_2\frac{d^2\beta_2}{d\omega_2^2}\Big/\xi_1\frac{d^2\beta_1}{d\omega_1^2}$

The phase modulation due to RF signal is given by the following expression:

$$\Delta\varphi(\tau)=\frac{\pi V_0}{V_\pi}\cos\omega_m\tau \tag{13}$$

where V_0 is the peak modulation voltage, ω_m is the angle frequency of RF signal and V_π is the half-wave voltage of the phase modulator. Then we derive the Taylor-series expansion of Eq.(13)

$$\Delta\varphi(\tau)\approx\frac{\pi V_0}{V_\pi}\left(1-\frac{\omega_m^2\tau^2}{2}+\frac{\omega_m^4\tau^4}{24}\right) \tag{14}$$

In the condition of $|\omega_m\tau|\leqslant\frac{1}{2\pi}$, $\frac{\omega_m^4\tau^4}{24}$ can be ignored, then, Eq.(14) can be expressed in a more convenient form:

$$\Delta\varphi(\tau)\approx\frac{\pi V_0}{V_\pi}\left(1-\frac{\omega_m^2\tau^2}{2}\right) \tag{15}$$

From Eq.(15) it is found that quadratic phase modulation is realized to form a time lens. And the time aperture of phase modulator based time lens can be obtained from the inequality $|\omega_m\tau|\leqslant\frac{1}{2\pi}$:

$$\tau_a=\frac{1}{2\pi\omega_m} \tag{16}$$

For a phase modulator based time lens, input pulse width should be smaller than this time aperture. For practical applications, input pulse with smaller than a small fraction of the signal period (around 15%) can be operated by the time lens, otherwise, approximation of quadratic phase modulation is not valid more. However, by single modulator driven with three harmonics of the clock frequency, truly parabolic modulation over a time aperture that extends across 70% of the period can be achieved.

Then, the focal time of phase modulator based time lens is:

$$f_T=\frac{\omega_0}{\Gamma_0\omega_m^2} \tag{17}$$

where Γ_0 is peak phase deviation (phase modulation depth) in the absence of velocity walk off:

$$\Gamma_0\equiv\frac{\omega_0\Delta n_0\xi}{c}=\pi\frac{V}{V_\pi} \tag{18}$$

where Δn_0 is the peak index of refraction change, and ξ is the total propagation distance.

Introduce Eq.(18) to Eq.(17), we get the focal time, just like focal length for a spatial lens.

$$f_T=\frac{\omega_0V_\pi}{\pi\omega_m^2V} \tag{19}$$

Accordingly, the f-number can be written as:

$$f_T^{\#}=\frac{\omega_0}{\Gamma_0\omega_m}=\frac{\omega_0V_\pi}{\pi\omega_m^2V} \tag{20}$$

We can substitute the f-number and obtain the resolution

$$\delta\tau=T_0f_T^{\#}=\frac{1}{\Gamma_0f_m} \tag{21}$$

where f_m is the frequency of the RF signal: $f_m=\omega_m/2\pi$. From Eq. (21) it is clearly that in order to improve the resolution of the phase modulator based time lens, peak modulation and modulating frequency need to be increased.

Phase modulator based time lens are limited by the finite time aperture induced by quadratic phase modulation approximation and the lower resolution due to the lower maximum phase modulation depth ($\leqslant10\pi$ as a typical value). However, phase modulator is easy to be realized and this prototype of time lens can suggest more implementation of time lens.

2.2 Cross phase modulation (XPM)

Time lens can be achieved using XPM between the input pulses and a parabolic pulse with different wavelength in a nonlinear medium such as high nonlinear fiber (HNLF) to induce a rigorous required quadratic phase shift, which is illustrated in Fig.3. Parabolic pulses can be generated in a passive and stable manner by a super-structured fiber Bragg grating.

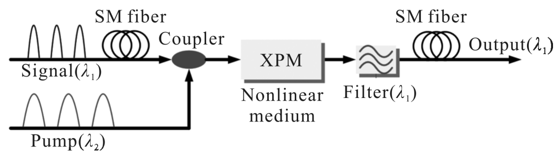


Fig.3 Schematic illustration of time lens based on XPM (SM: single mode)

Due to the phase function (4) and temporal imaging condition Eq. (7), the quadratic phase modulation for a time lens can be expressed as:

$$\varphi_1(\tau)=\frac{\omega_0\tau^2}{2f_T}=-\frac{1}{2}\left(\frac{1}{\xi_1\frac{d^2\beta_1}{d\omega_1^2}}+\frac{1}{\xi_2\frac{d^2\beta_2}{d\omega_2^2}}\right)\tau^2 \tag{22}$$

While, the phase shift generated by XPM with parabolic pulse is

$$\varphi_2(\tau)=-2l_0\gamma_0|A_p(\tau)|^2 \tag{23}$$

where l_0 and γ_0 are the length and nonlinear coefficient of HNLF, respectively. $A_p(\tau)$ is the envelope of the pump pulse. In order to form a time lens $\varphi_1(\tau)=\varphi_2(\tau)$ should be achieved, so

$$|A_p(\tau)|^2=\frac{1}{4l_0\gamma_0}\left(\frac{1}{\xi_1\frac{d^2\beta_1}{d\omega_1^2}}+\frac{1}{\xi_2\frac{d^2\beta_2}{d\omega_2^2}}\right)\tau^2=-a\tau^2 \tag{24}$$

and

$$a=-\frac{1}{4l_0\gamma_0}\left(\frac{1}{\xi_1\frac{d^2\beta_1}{d\omega_1^2}}+\frac{1}{\xi_2\frac{d^2\beta_2}{d\omega_2^2}}\right) \tag{25}$$

where $\xi_{1,2}$ and $d^2\beta_1/d\omega_1^2$, $d^2\beta_2/d\omega_2^2$ are length and dispersion coefficient of input and output fiber, respectively. In practical applications, the envelope of parabolic pulse is usually with a constant b

$$|A_p(\tau)|^2=-a\tau^2+b \tag{26}$$

The existence of b presents a time-independent phase, however, there is no influences on the envelope of signal pulse. In order to realize the phase modulation during the while of width of signal pulse, it is required that

$$\tau_p \geqslant \tau_s \tag{27}$$

where τ_p and τ_s are temporal width of pump and signal pulses, respectively.

From Eqs.(22) and (25), we can get the focal time of time lens based on XPM by assuming b=0:

$$f_T=\frac{\omega_0}{4l_0\gamma_0 a} \tag{28}$$

We consider τ_p is the effective time aperture due to Eq.(25), the f-number can be written as:

$$f_T^{\#}=\frac{\omega_0}{4\tau_p l_0\gamma_0 a} \tag{29}$$

Thus, the resolution is:

$$\delta\tau=\frac{\pi}{\tau_p 2l_0\gamma_0 a} \tag{30}$$

From Eq.(30) it is found that the resolution of time lens based on XPM is limited by the temporal width of the parabolic pump. Therefore, improving the resolution of time lens based on XPM can be realized by generation and broadening the pump pulse.

2.3 Sum-frequency generation (SFG)

Time lens can also be achieved by a parametric process, such as three-[11] or four-wave mixing[13-14] (TWM or FWM). When a linearly chirped pump pulse is mixed with a signal pulse in either a $\chi^{(2)}$ or $\chi^{(3)}$ parametric process, the quadratic phase modulation can be imparted on the converted pulse. For TWM based time lens, implementation by SFG is analyzed as follow and the DFG is similar.

In a SFG based time lens, input pulse is mixed in a nonlinear crystal with a linearly chirped pump pulse, under ideal phase matching conditions, the SFG process will introduce a signal $A_2(\tau)=Ap(\tau)A_1(\tau)$ at a new carrier frequency $\omega_2=\omega_1+\omega_p$, which is illustrated in Fig.4. The phase shift of output pulse can be written

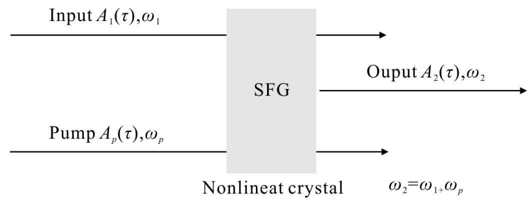


Fig.4 Schematic illustration of a SFG based time lens

by an equivalent frequency chirp as:

$$d^2\varphi(\tau)/d\tau^2=d\omega_1(\tau)/d\tau \tag{31}$$

where $\varphi(\tau)$ is the time lens phase function. We may also express this process with a physical focal length ξ_f or focal group delay dispersion (GDD) as:

$$\varphi_f''=\xi_f\beta''=-\left[d\omega_1(\tau)/d\tau\right]^{-1} \tag{32}$$

where $\beta''=d^2\beta(\omega)/d\omega^2$ is the group velocity dispersion (GVD). Then, we get the phase function and focal time as:

$$\varphi(\tau)=-\frac{\tau^2}{2\xi_f\beta''} \quad \text{and} \quad f_T=-\omega_0\xi_f\beta'' \tag{33}$$

The f-number of the time lens and thus the resolution of the temporal imaging system are inversely proportional to the pump pulse bandwidth $\Delta\omega_p$ imparted by the parametric process

$$f_T^{\#}=\omega_2/\Delta\omega_p \tag{34}$$

Generally, the pump pulse employed in SFG based time lens is initially an ultra-short transform-limited Gaussian pulse with a FWHM $\Delta\tau_{p0}$. Propagating the pulse through a GDD φ_p'' creates a chirped pump pulse with a temporal width

$$\Delta\tau_p=\Delta\tau_{p0}\sqrt{1+(\varphi_p''/\varphi_{Rp}'')^2} \tag{35}$$

and the chirp rate is:

$$d\omega_1(\tau)/d\tau=\left\{\varphi''\left[1+(\varphi_{Rp}''/\varphi_p'')^2\right]\right\}^{-1} \tag{36}$$

where $\varphi_{Rp}''=\Delta\tau_{p0}/(4\ln 2)$ is the Gaussian dispersion that is the GDD to broaden the pulse by $\sqrt{2}$. It can also be expressed in terms of the transform-limited pulse width and bandwidth as $\varphi''_{Rp}=\Delta\tau_{p0}/\Delta\omega_p$. For large stretching, this produces a time lens with a width $\Delta\tau_p\approx\Delta\tau_{p0}|\varphi''_p/\varphi''_{Rp}|=|\varphi''_p|\Delta\omega_p$ and focal GDD $\varphi''_f\approx-\varphi''_p$.

Temporal imaging condition and magnification factor for SFG based time lens can be rewritten as:

$$\frac{1}{\varphi''_1}+\frac{1}{\varphi''_2}=\frac{1}{\varphi''_f} \text{ and } M=-\varphi''_2/\varphi''_1 \tag{37}$$

For an ideal imaging system with large magnification $|M|\gg1$ and this Gaussian pump pulse profile, there is a Gaussian impulse response with width $\delta\tau_{out}$, which, when referred to the input, results in a resolution $\delta\tau_{in}$ that is equal to the undispersed pump pulse width:

$$\delta\tau_{in}=\delta\tau_{out}/|M|=\Delta\tau_{p0} \tag{38}$$

Using description of f–number by Eq.(34), Eq.(38) can be expressed in a practical form as follows

$$\delta\tau_{in}=0.44T_2f^{\#}_T \tag{39}$$

where T_2 is an optical period of the output pulse.

When $|M|\ll1$, the limitation on the resolvable feature that can be generated at the output of the system is also equal to the undispersed pump profile

$$\delta\tau_{in}=\delta\tau_{out}/|M|=\Delta\tau_{p0}|M| \tag{40}$$

Here resolution described by Eqs.(39) and (40) does not include the influences of the group velocity mismatch (GVM). An extensive analysis on GVM in SFG time lens has been presented in Ref.[11]. The general result including both of these effects is summarized: GVM in the nonlinear crystal produces a filtering effect in the temporal imaging system. The bandwidth of the filter decreases as the interaction length is increased to get maximum conversion efficiency, thus, there is a tradeoff between the strength of the parametric output and the temporal performance (mainly described by resolution).

For the TWM based time lens, there are several limitations for its further improvement as follows:

(1) SFG and DFG occur only in materials with a second–order non linearity, which limits choosing material for improved performances;

(2) Remarkable wavelength difference between output and signal pulses limits some applications that require a wavelength maintenance, especially in telecommunications;

(3) When a high power pump is employed, the parametric process is affected by the nonlinear absorption;

(4) Due to influences of GVM, conversion efficiency and resolution can not be increased simultaneity.

2.4 Four–wave mixing (FWM)

Another parametric process for producing a time lens is FWM as shown in Fig.5. Unlike SFG and DFG that occur only in materials with second–order nonlinearity, FWM occurs in materials including silica glass and silicon. Therefore, other classes of optical devices with mature fabrication processes, including fibers and silicon–on insulator devices, can be used for FWM time lens. In addition, the converted wavelength in the FWM process is generated at nearby input wavelength.

In the parametric time–lens scheme, the bandwidth of the conversion process determines the temporal resolution of the imaging system.

This dispersion tailoring allows for conversion bandwidths as large as 150 nm, which enables the characterization of single transient phenomena or rapidly changing waveforms with femto–second resolution.

The phase function $\varphi_f(\tau)$ of FWM based time lens is:

$$\varphi_f(\tau)=\frac{\tau^2}{2\varphi''_f} \tag{41}$$

where φ''_f is the focal GDD associated with the lens and is equal to the inverse of the second derivative of the phase.

The dispersive elements before and after the lens are characterized by their GDD parameters $\phi''_1=\beta_2^{(1)}L_1$ and $\phi''_2=\beta_2^{(1)}L_2$, here $\beta_2^{(1,2)}$ and $L_{1,2}$ are GVD and the length dispersion of dispersion, respectively. Similar to

SFG and DFG, the temporal imaging condition and magnification factor of FWM based time lens are with the same as that of SFG implementation described by Eqs.(36) and (37), respectively.

As shown in Fig.5, a Gaussian pump pulse propagates through a dispersive medium with a much

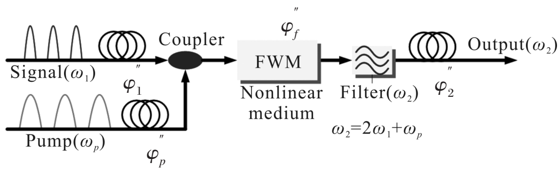


Fig.5 Schematic illustration of a FWM based time lens

longer than dispersion length of pulse. Then, the pump pulse is temporally broaden and linearly chirped with a quadratic phase as:

$$\varphi_p(\tau)=-\frac{\tau^2}{2\varphi''_p}=-\frac{\tau^2}{2\beta_2^{(p)}L_p} \tag{42}$$

Where $\varphi''_p=\beta_2^{(p)}L_p$ is the GDD experienced by the pump, $\beta_2^{(p)}$ is the GVD, and L_p is the length of dispersive element for pump. If an input signal pulse with electric field amplitude $E_s(\tau)$ is mixed with the chirped pump $E_p(\tau)$ via FWM process, the idler is:

$$E_s(\tau)\propto E_p^{(2)}E_s^*(\tau) \tag{43}$$

Then, the quadratic phase modulation described by Eq.(41) is added to signal pulse. The focal GDD is:

$$\varphi''_f=-\varphi''_p/2 \tag{44}$$

Then, the phase function of FWM based tine lens can be expressed as:

$$\varphi_f(\tau)=\frac{\tau^2}{\beta_2^{(p)}L_p} \tag{45}$$

The focal time is:

$$f_T=\omega_0\beta_2^{(p)}L_p/2=\omega_0\beta_2^{(p)}L_s \tag{46}$$

where L_s is the length of dispersive element for signal, and GVD is considered as the equal for both signal and pump. When a FWM based time lens is used for time to frequency conversion, the time-to-frequency conversion factor can be calculated by Eqs.(12) and (46) by:

$$\Delta t/\Delta\omega=-\beta_2^{(p)}L_p/2=-\beta_2^{(p)}L_s \tag{47}$$

where Δt is the temporal shift of the input signal, and $\Delta\omega$ is the resulting spectral shift. Therefore, by converting a narrow-band signal over twice the pump bandwidth, the approximate record length can be defined as:

$$\tau_{\text{record}}=2\beta_2^{(p)}L_s\Delta\omega_{\text{pump}} \tag{48}$$

where $\Delta\omega_{\text{pump}}$ is the spectral width of pump pulse. The pump-pulse bandwidth and the length of the dispersive path determine the record length. The resolution is given by

$$\tau_{\text{resolution}}=\tau_{\text{pump}}/\sqrt{2} \tag{49}$$

where τ_{pump} is the pump pulse width, which limits the resolution in principle.

FWM based time lens is achieved for impressive results, however, for telecommunications band, there are significant influences of some competing nonlinear effects, such as SPM, XPM, dispersion and nonlinear absorption, which compromises the temporal response of time lens. These problems can be avoided by restricting the pump power, and might be further alleviated by switching to other waveguides that shift the absorption edge to lower wavelengths. There is no doubt that FWM approach tends to attract most attentions of researchers for development of time lens.

3 Applications in ultra-fast pulse measurement

With the improved data rates of next generation telecommunication and development in ultra-fast chemical[15] and physical phenomena[16], it has become important to develop techniques that enable simple measurements of optical pulses with sub-picosecond resolution. However, achieved resolution is currently limited to several picoseconds resolution based on current photo detectors, oscilloscopes, and streak cameras. Time lens can be utilized in pulse magnification and time to frequency conversion to beat the bottleneck of ultra-fast pulse measurement.

When used for pulse magnification, a time lens is just called a time microscope, which can expand the temporal waveform w hile preserving the overall envelope profile. By changing the time scale, this technique allows direct measurement of ultra-fast pulse with current available instruments. When utilized for time to frequency conversion, time lens can be considered as a Fourier transformer. The waveform of ultra-fast pulse can be obtained from the frequency spectra of output pulse by current OSA.

Resolution and record length are employed to describe the shortest and longest pulse of the ultra-fast pulse measurement system as the mainly practical performances. Resolution and record length of ultra-fast pulse measurement realized by time lens are illustrated in Fig.6. FWM based approach presents 220 fs resolution and 100 ps record length, however, future ultra-fast chemical and physical applications requires a ~10 fs resolution[15-16].

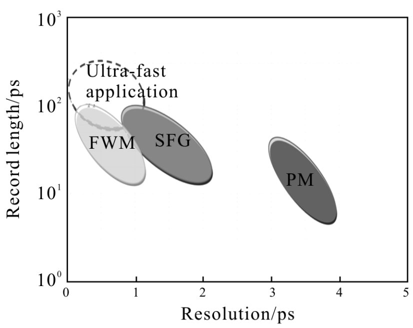


Fig.6 Resolution and record length of different implementations of time lens and ultra-fast applications

4 Potential development of time lens

In general, current FWM based time lens can not reach the requirement of further applications in ultra-fast chemistry and physics. For this reason, there is a strong motivation to explore novel material platforms that exhibit high nonlinearity with weak competing nonlinear effects to optimize the resolution of time lens. Recently, nano-metallic material and Graphene are found to be with extra-strong optical nonlinearity, which can be consider as significant opportunities to realize novel time lens with improved performances.

Metal composite materials, or nanostructured metal surfaces such as nano-rod and nano-periodic structure exhibit enhanced hight nonlinear susceptibility owing to extreme subwavelength focusing by surface plasma. Requirements for signal and pump pulse and the optimized technical design are still need to be contributed in this field[17-18].

Hendry et al. [19] have recently reported that Graphene possesses an extra-strong third-order optical nonlinearity ($|\chi^{(3)}| \sim 10^{-7}$ electrostatic units) which could have some potential applications such as wavelength converters, stabilizing multiwavelength oscillation and THz devices. Graphene can generate strong FWM in low-power pump condition (mW level) using only a piece of ultra-thin Graphene. However, owing to the simultaneous linear and nonlinear absorption, it is very hard to obtain the idler pulse output from process of FWM in Graphene. This is the mainly limitation of Graphene based time lens[20].

5 Conclusion

"The powerful concept of the time lens is perhaps finally coming of age." Just like David J. Richardson presented in Ref. [6], it is the time to consider time lens as not only an interesting optical instrument but also a bottle-neck-breaking solution of next generation telecommunication and ultra-fast pulse measurement. By applying techniques from Photonics, realizing and improving time lens are realized by phase modulator, SPM, XPM, SFG, DFG and FWM. Due to these approaches time lens are utilized for different applications in shaping, magnification, compression of pulses and ultra-fast pulse measurement. For further ultra-fast chemistry and physics, resolution of temporal imaging system is the bottle-neck. We believe that there is a wealth of time lens based on surface plasmon and Graphene enhanced nonlinearity still to be made, limited only by the creativity of the inventor.

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